Parallel Large Scale Feature Selection for Logistic Regression

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SIAM Data Mining, 2009

Outline

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Logistic Regression

$$P(y = 1 \mid \vec{x}_i, \vec{\beta}) = \frac{e^{\vec{\beta}\cdot\vec{x}}}{1 + e^{\vec{\beta}\cdot\vec{x}}}$$

$$\vec{\beta} = \underset{\vec{\beta}}{\text{argmax}} \sum_{i=1}^{N} \Big(y_i \ln p_i + (1 - y_i) \ln(1 - p_i) \Big)$$

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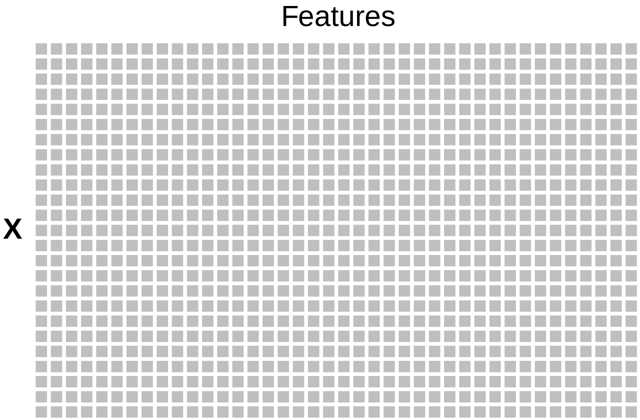
Feature Selection



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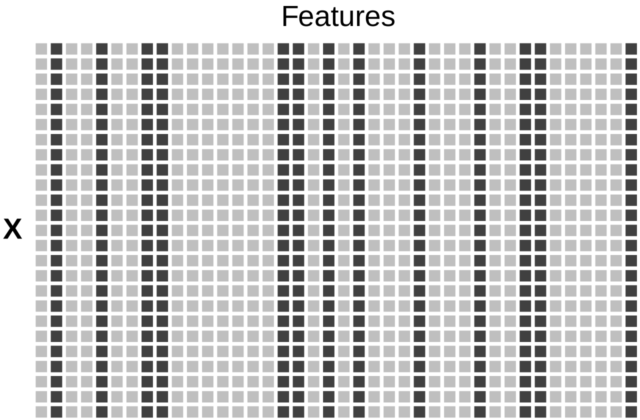

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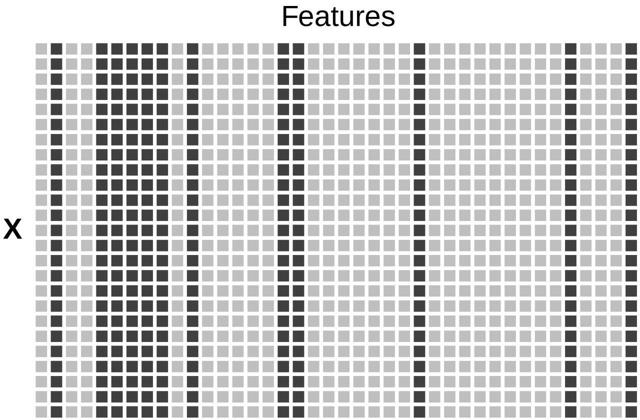

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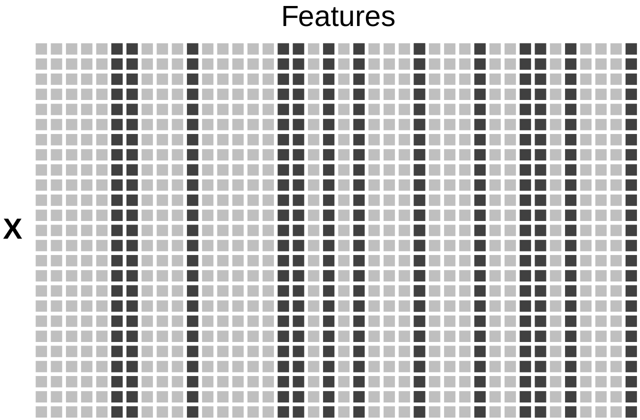
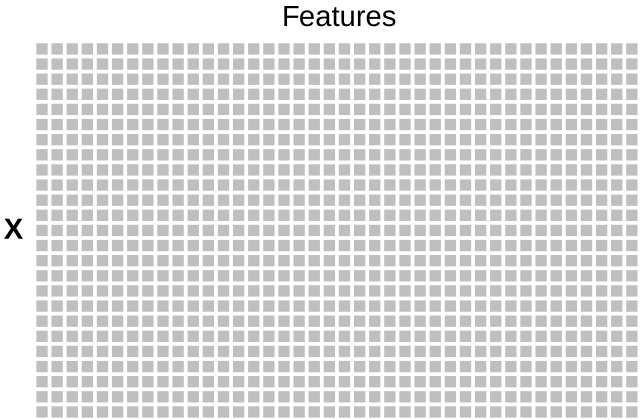


Feature Selection

For D features, train the model $O(2^D)$ times

Forward Feature Selection



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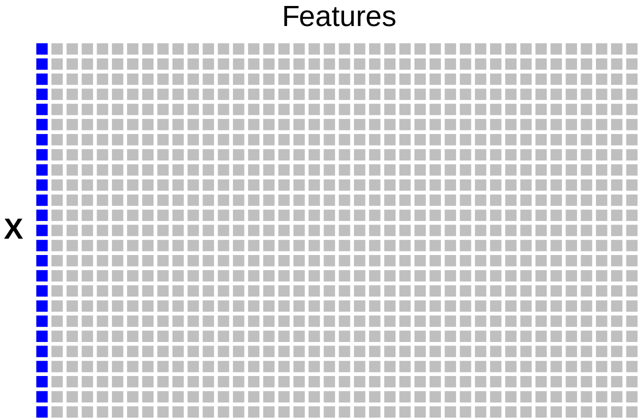

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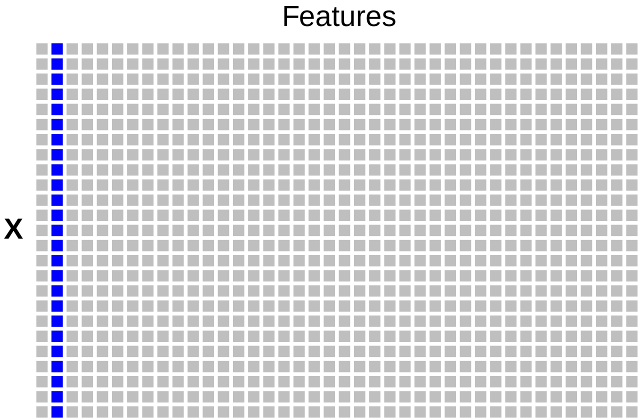

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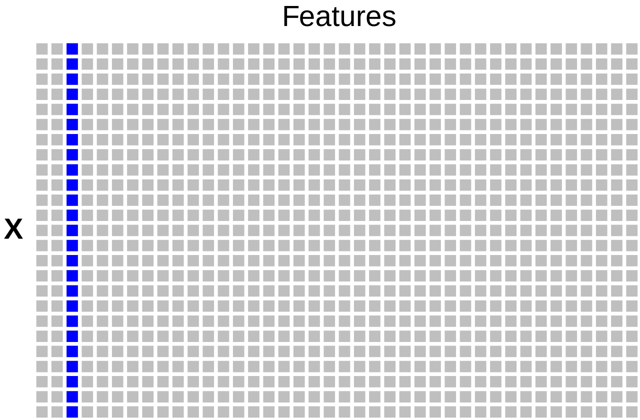

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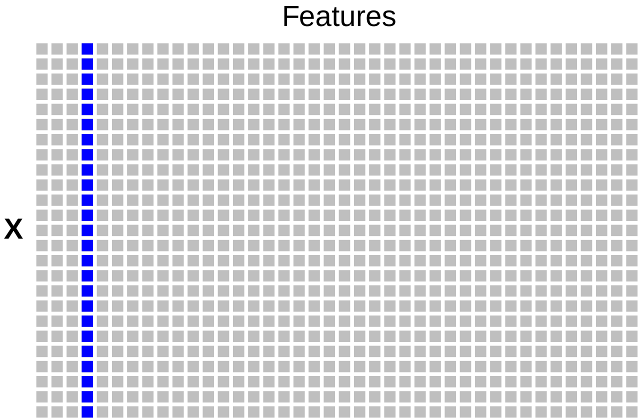

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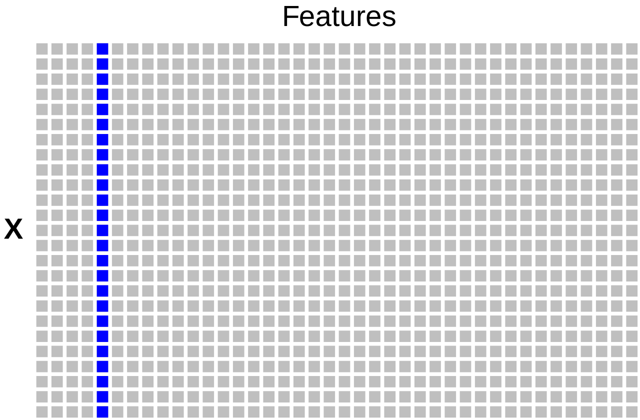

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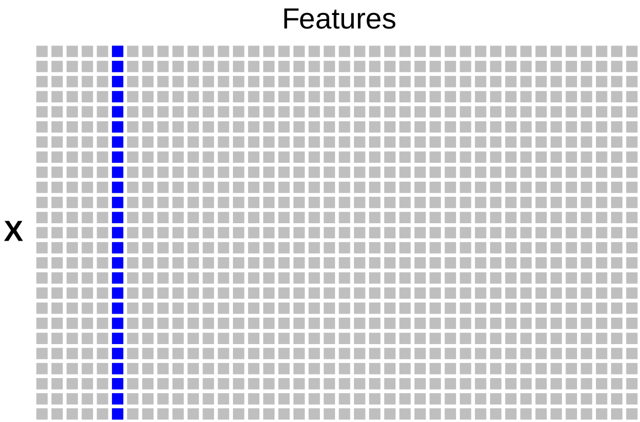

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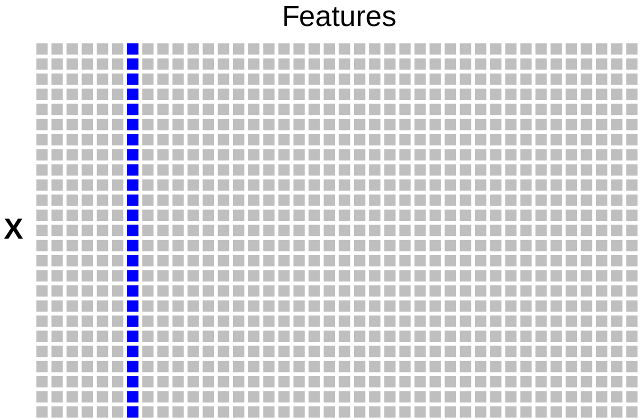

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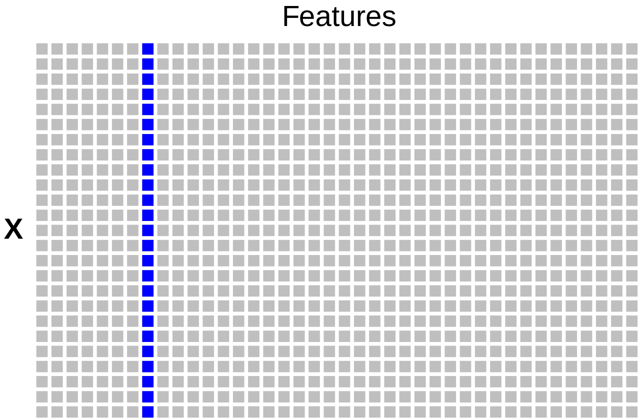

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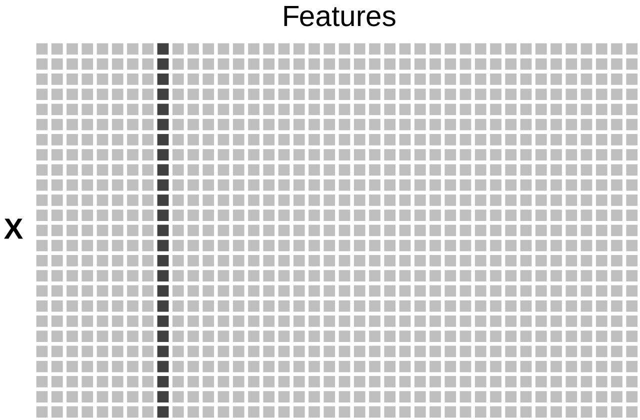

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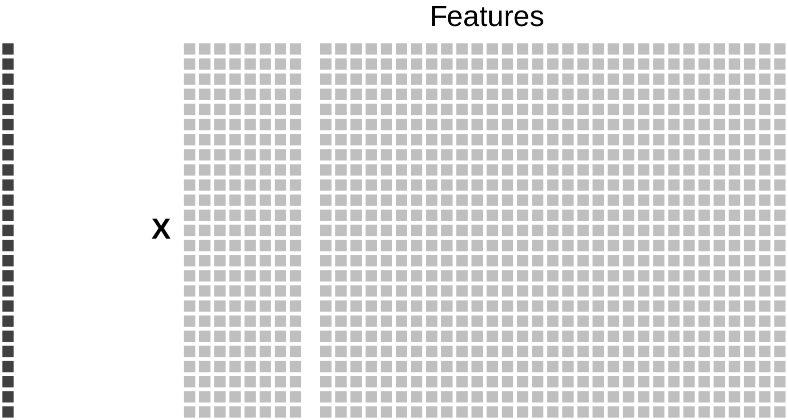

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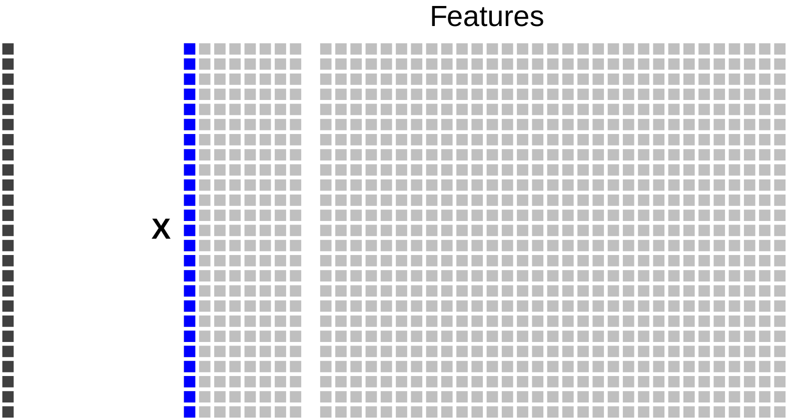

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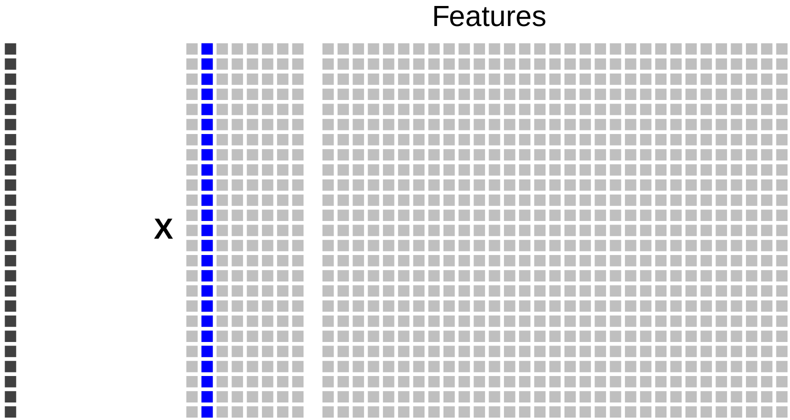

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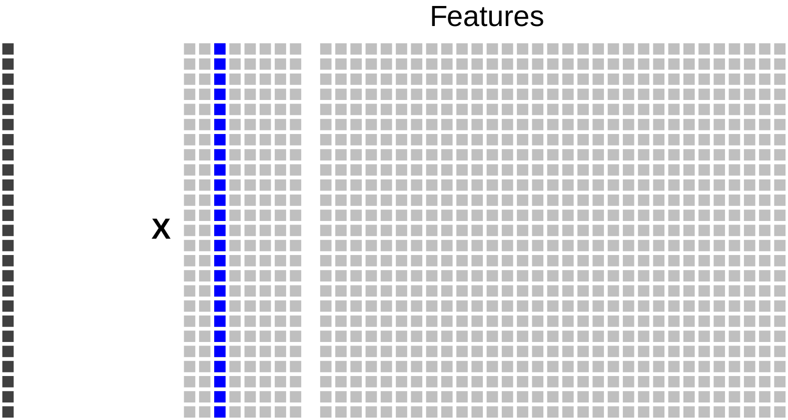

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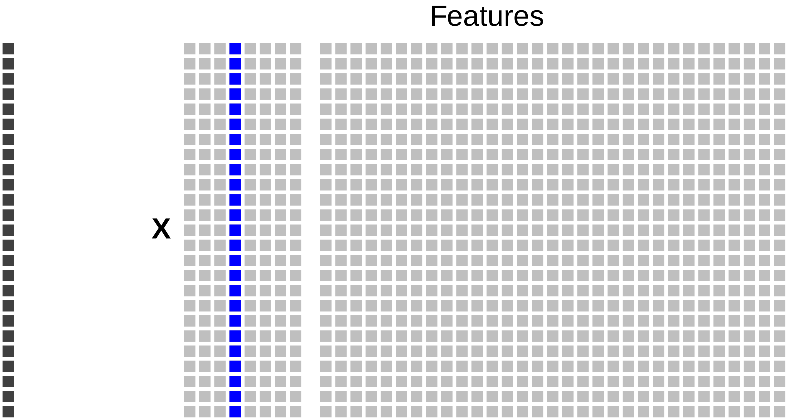

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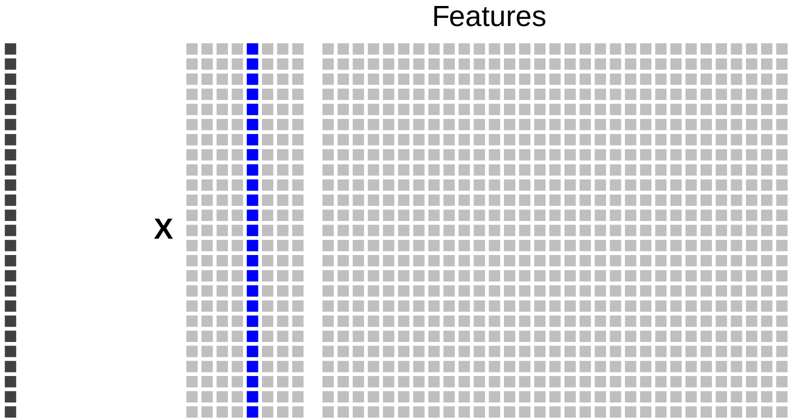

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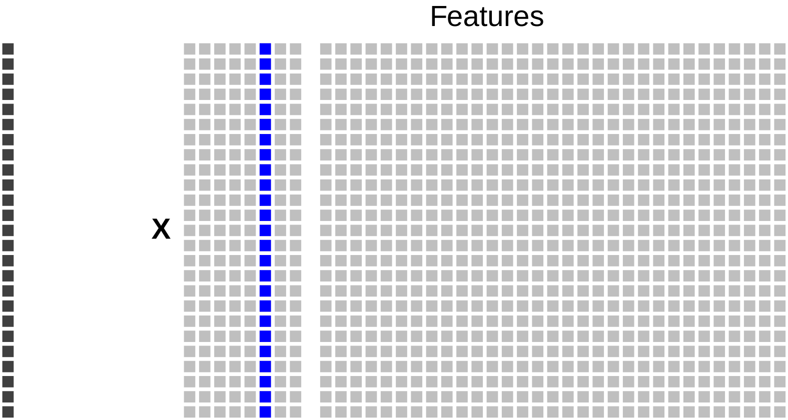

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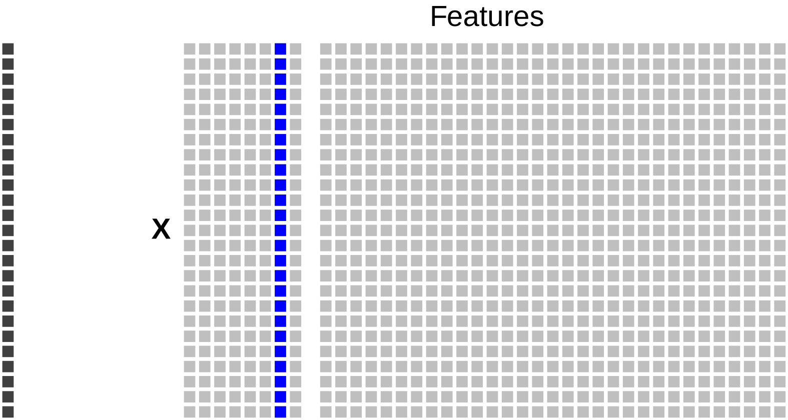

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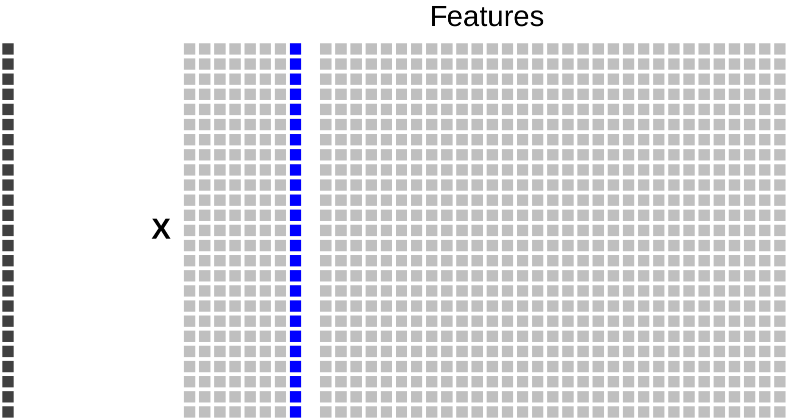

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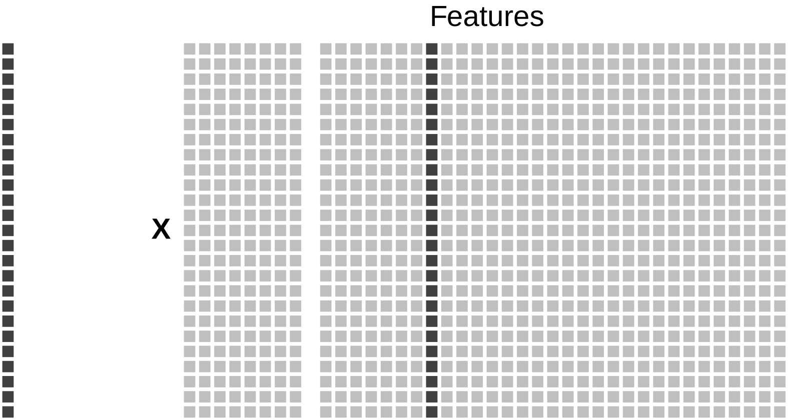

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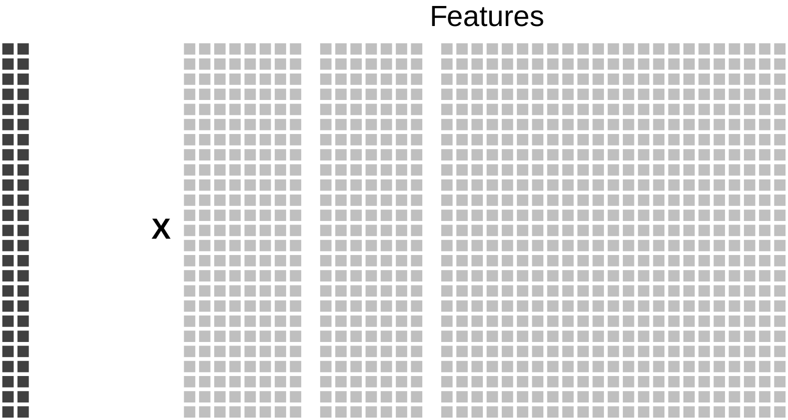

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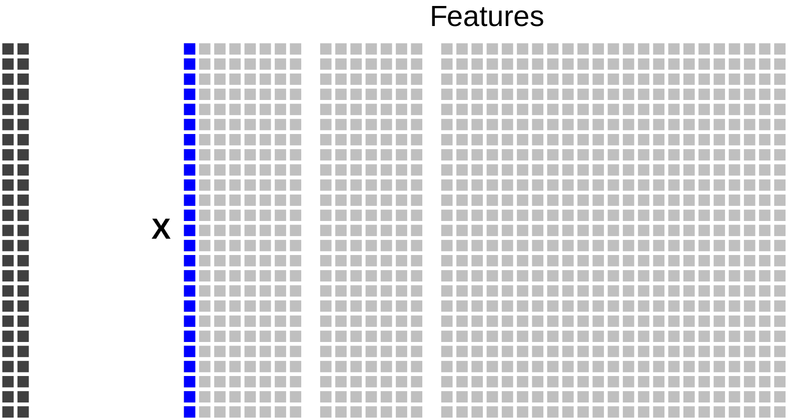

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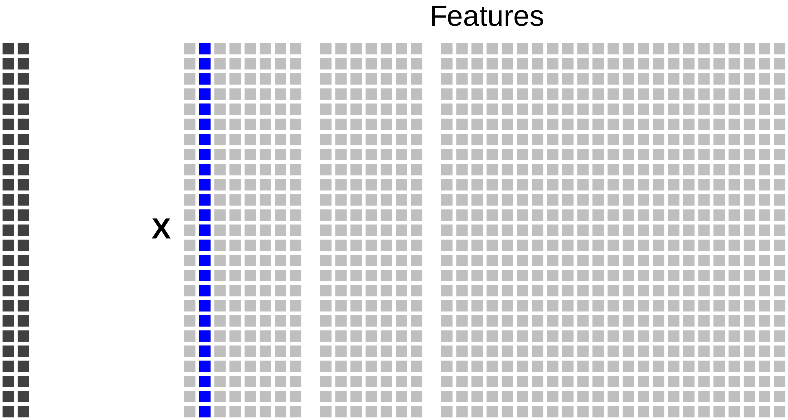

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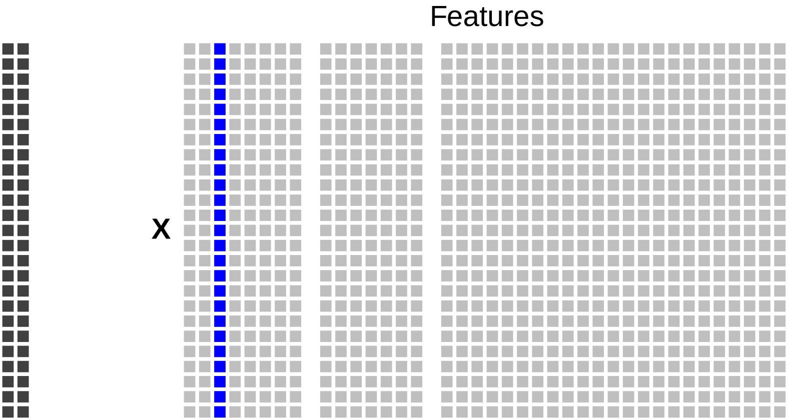

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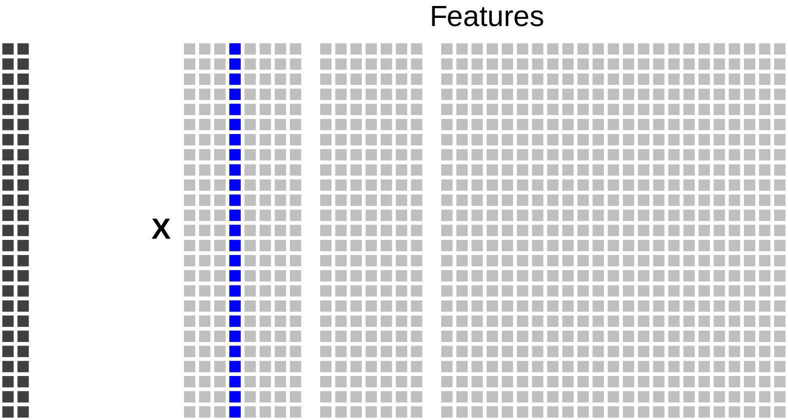

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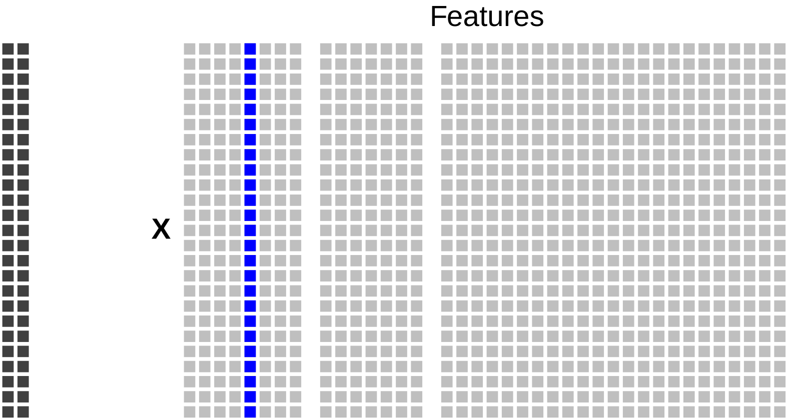

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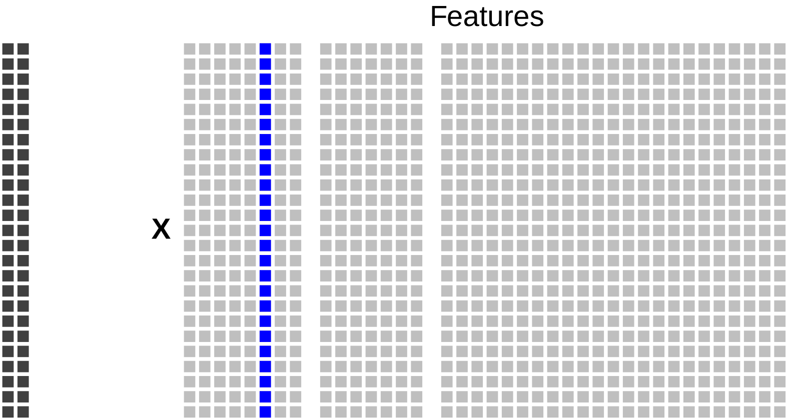

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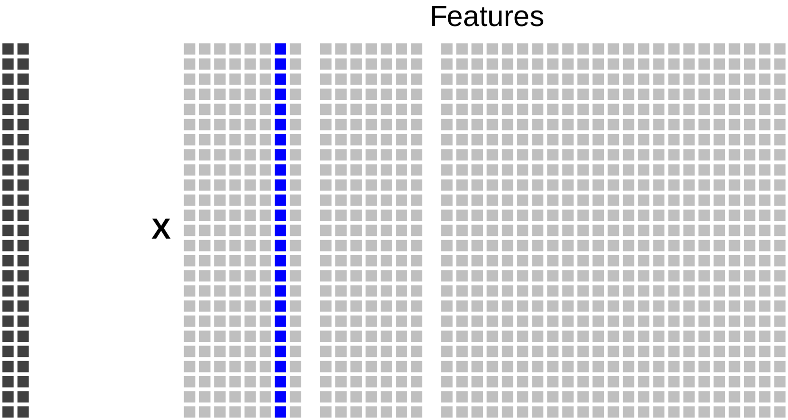

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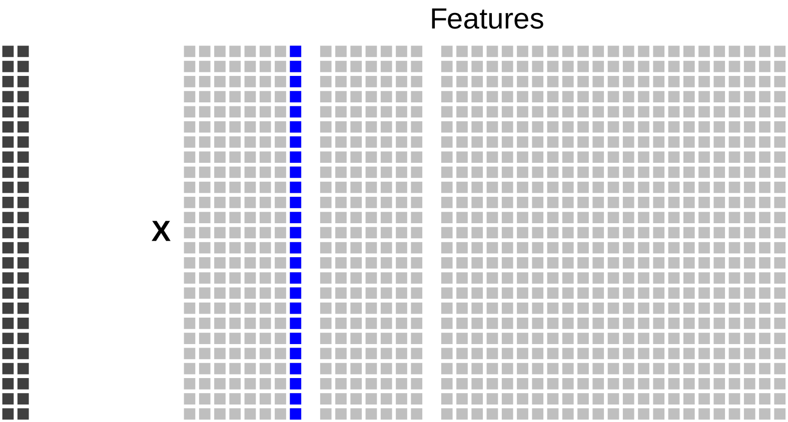

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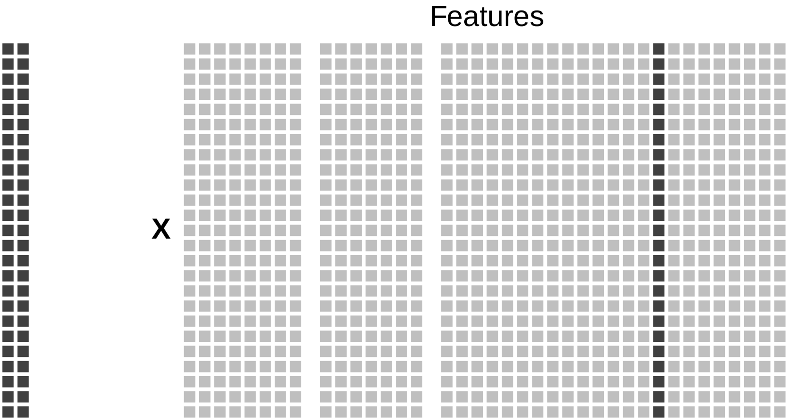

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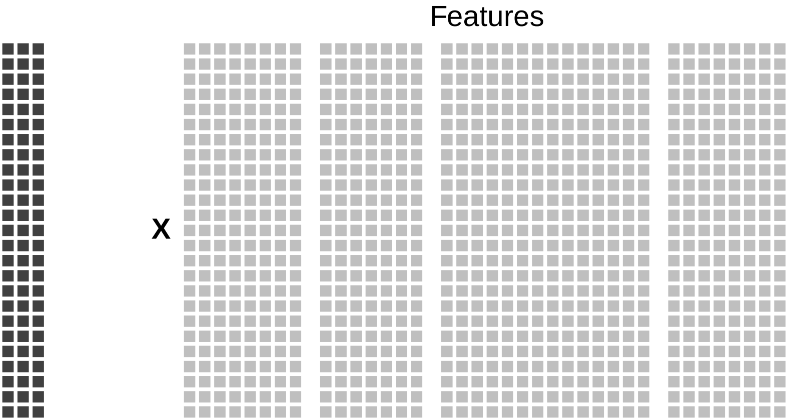

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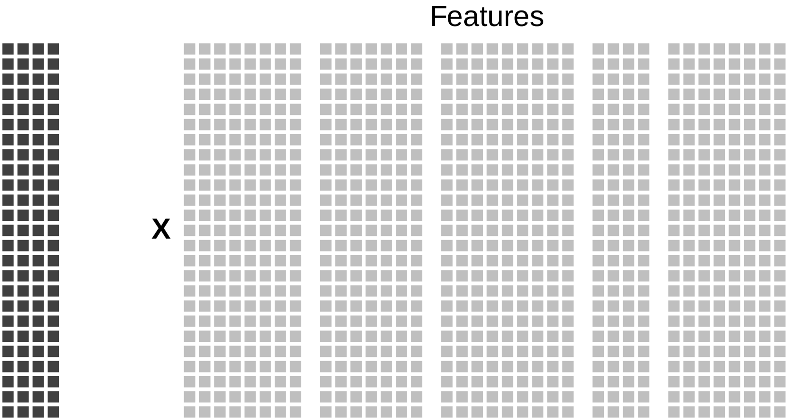

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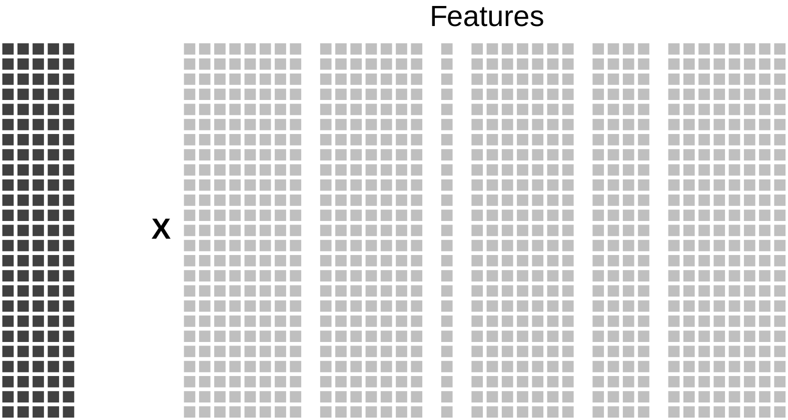

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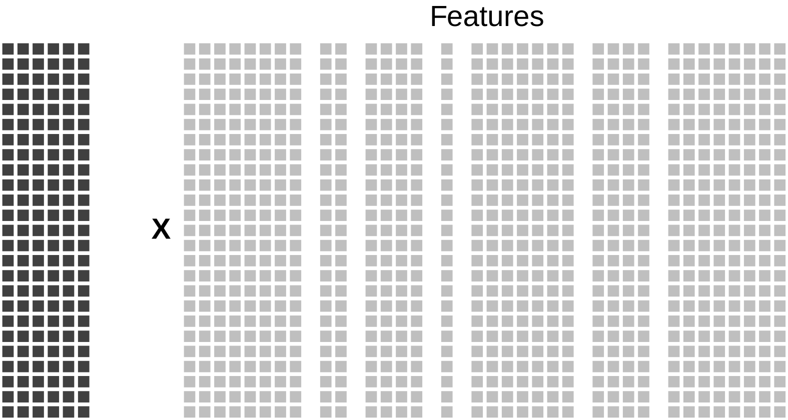

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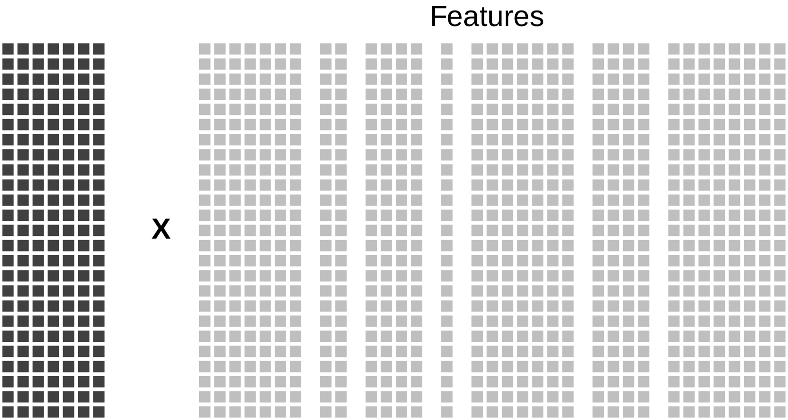
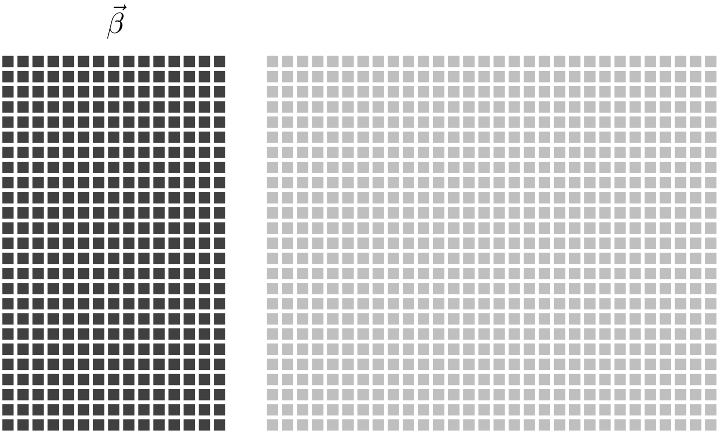


Forward Feature Selection

For D features, train the model $O(D^2)$ times

Outline

Single Feature Optimization



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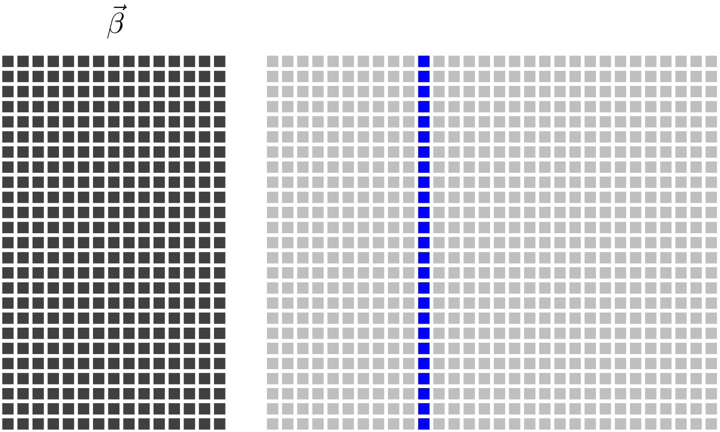

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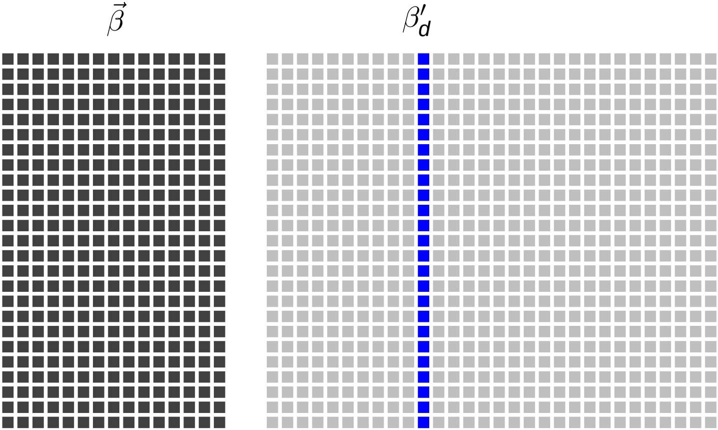

Newton's Method

$$p_{id} = \frac{e^{\vec{\beta}\cdot\vec{x}_i + x'_{id}\beta'_d}}{1 + e^{\vec{\beta}\cdot\vec{x}_i + x'_{id}\beta'_d}}$$

$$\beta'_d = \operatorname*{argmax}_{\beta'_d} \sum_{i=1}^{N} \Big(y_i \ln p_{id} + (1 - y_i) \ln(1 - p_{id}) \Big)$$

Newton's Method

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$$\frac{\partial L}{\partial \beta'_d} = \sum_{i=1}^{N} x'_{id}(y_i - p_{id})$$

$$\frac{\partial^2 L}{\partial \beta'^2_d} = -\sum_{i=1}^{N} p_{id}(1 - p_{id})x'^2_{id}$$

Histogram Approximation

- As N grows, Newton's method slows down considerably
- B bins, based on predicted probability of *base* model
  - using only $\vec{\beta}$ and $\vec{x}$
- Newton's method dependent on B instead of N
  - $N >> B$

Map Reduce implementation

- Map: Parallel over *records*
  - **Input:** Base features $\vec{x}_i$, class y_i, new features $\vec{x}_i'$
  - Predict using the base model p_i
  - **Output:** $(x_{id}', \langle y_i, p_i \rangle)$ for every feature x_{id}' in $\vec{x}_i'$
- Reduce: Parallel over *features*
  - **Input:** $x_d', \langle y_i, p_i \rangle^n$
  - Use Newton's method to find β_d' that maximizes scoring measure
  - With or without histogram approximation
  - **Output:** Estimated coefficient β_d'

Evaluate the coefficients on test dataset to evaluate utility

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Outline

Methods

- IRLS: Iteratively Re-weighted Least Squares
  - P. Komarek and A. Moore, *ICDM 2005*[1]
  - Fast, efficient single machine implementation of Logistic Regression
  - Retrain classifier for each candidate feature
- SFO: Single Feature Optimization
  - Use IRLS to train the "base" model
- GD: Gradient Method
  - S. Perkins and J. Theiler, *ICML 2003*
  - Ranks features according to their gradient on training data
  - Parallelize it same way as SFO

[1] http://www.autonlab.org/autonweb/10538.html

Mushroom Dataset

Base Features	Feature Class	IRLS -LL	SFO -LL	SFO Rank	GD Rank
bias	odor	0.111	0.076	1	2
	spore-print-color	0.558	0.543	2	1
	gill-color	0.623	0.604	3	9
	stalk-surface-above	0.696	0.692	5	3
	ring-type	0.711	0.687	4	8
bias, odor	spore-print-color	0.074	0.069	1	5
	stalk-surface-above	0.098	0.090	3	3
	population	0.099	0.092	5	6
	gill-color	0.099	0.091	4	7
	stalk-color-below	0.100	0.086	2	4

Table: The negative test set log-likelihood for the top features in the Mushroom data set as selected by IRLS, the corresponding SFO scores, and rankings from SFO and the gradient method.

InternetAds Dataset

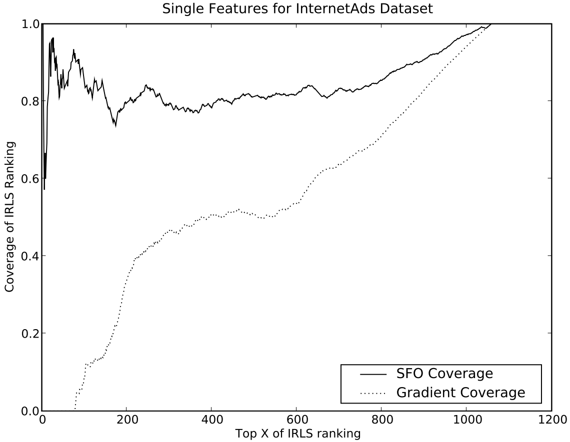

Figure: Coverage of the IRLS ranking by SFO and the Gradient method for the Internet Ads data. The features were ranked by test set log-likelihood.

RCV1

Round 1		Round 2		Round 3		Round 4		Round 5	
bias		bias econ		bias econ muni		bias econ muni defi		bias econ muni defi *shar*	
econ	283.7	muni	204.3	defi	110.2	*shar*	106.7	infl	79.5
defi	213.7	*shar*	139.3	*shar*	106.8	stat	82.1	wag	77.1
infl	190.1	coup	131.3	stat	90.2	wag	79.7	stat	76.5
gdp	182.9	obli	110.3	infl	87.2	*profit*	79.1	mood	68.6
muni	176.3	*prof*	106.1	gdp	86.6	infl	74.7	*dig*	66.0

Table: Top 5 features & estimated improvement on training set loglikelihood.

Timing Results

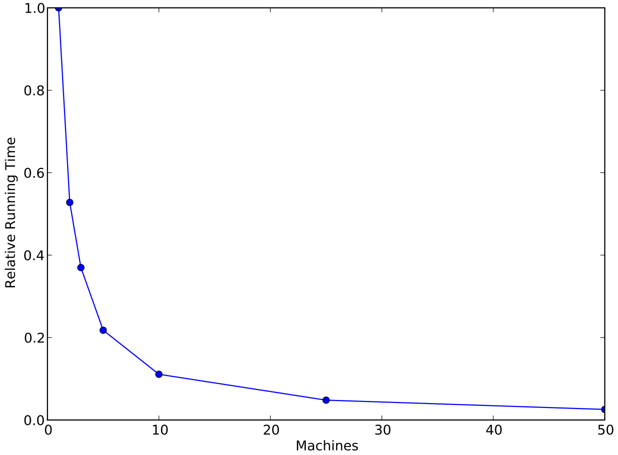

Figure: Timing (10,000,000 records / 100,000 features)

Speedup

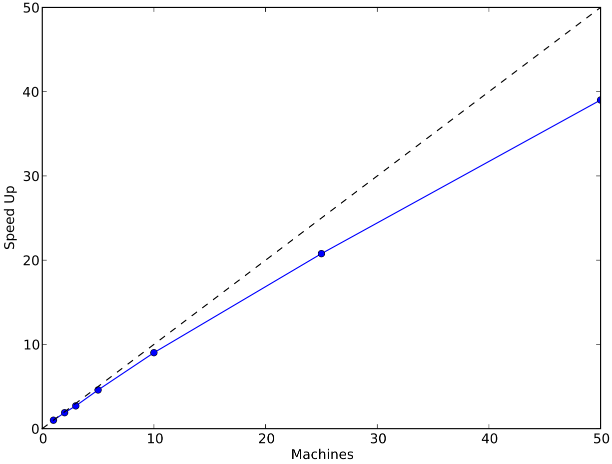

Figure: Speedup (10,000,000 records / 100,000 features)

Summary

- Introduce Single Feature Optimization (SFO)
  - approximation to Forward Feature Selection
- To scale to large datasets, utilize MapReduce for parallelism
- Histogram Approximation is used to further scalability

- Future Work:
  - Multiple Feature Optimization
    - pairs of features
  - Optimize on metrics other than LogLikelihood

Thank You

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SIAM Data Mining, 2009

Histogram Approximation

- For each bin b
  - Mean probability p_{id} of the bin $\hat{p}_b$
  - Total number of records in the bin N_b
  - Number of records in which $x_d = 1$, N_b^+
- Calculate p'_b using $\hat{p}_b$ and β_d

$$\frac{\partial L}{\partial \beta'_d} = \sum_{b=1}^{B} N_b^+ - p'_b \cdot N_b$$

$$\frac{\partial^L}{\partial \beta'^2_d} = -\sum_{b=1}^{B} N_b \cdot p'_b \cdot (1 - p'_b)$$

Map Reduce implementation

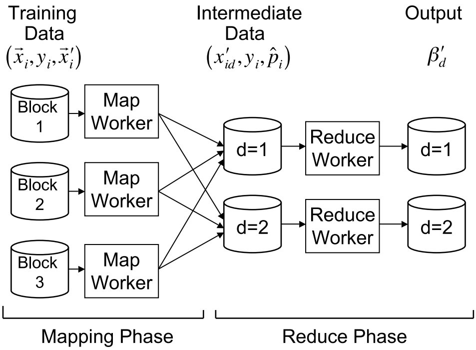

Figure: Map: operate on training data $(\vec{x}_i, y_i, \vec{x}_i')$ to produce intermediate records (y_i, p_i) for each new feature in the record $\vec{x}_i'$. Reduce: operate on intermediate records, computing coefficients for the new features β'_d.